Modeling spatially dependent shrub cover data in riparian zones

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Forest understory vegetation

- Critical components of forest ecosystems
- Contribute to biodiversity
- Protect against erosion
- Influence nutrient cycles
- Provide forage and cover for wildlife

→ Importance of modeling understory vegetation characteristics

Abundance measures

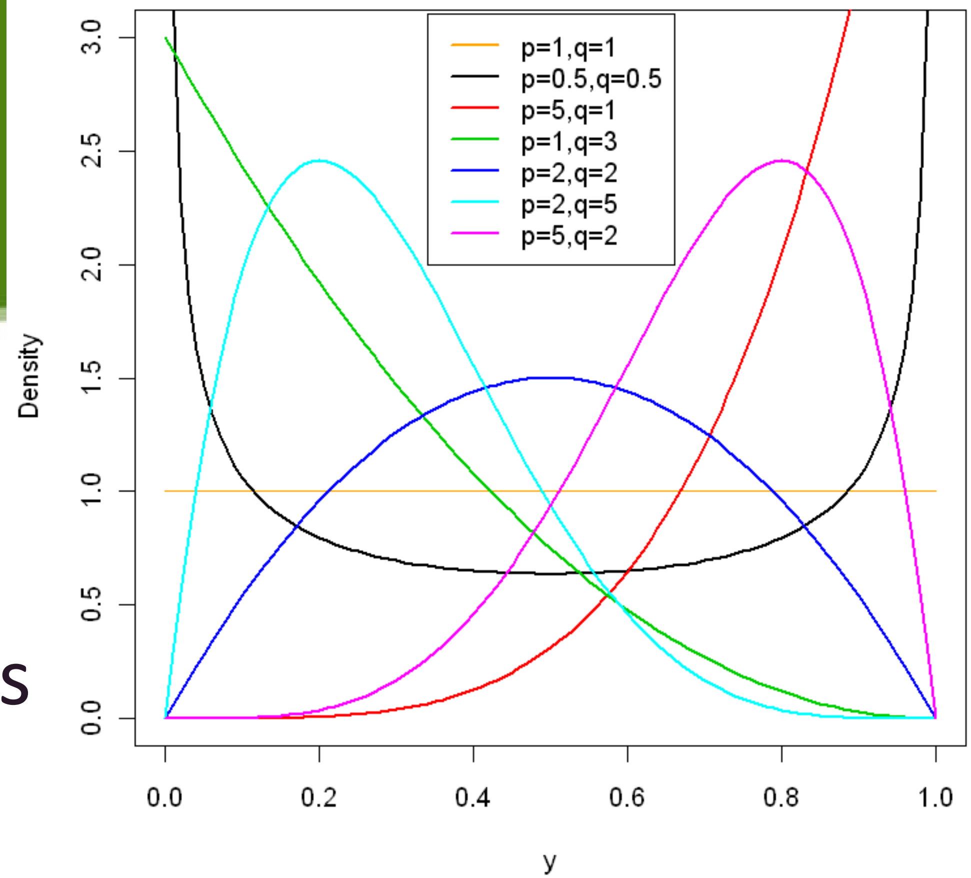
- Number of plant individuals
- Binary occurrences (presence/absence)
- Biomass per unit area
- Plant cover (e.g., percent shrub cover)
  - Prediction inherently difficult
  - Bounded between 0 and 1
  - Many zero observations & heteroscedastic error variance
  - Often subject to spatial dependence

→ Distributional features tend to be ignored in analysis

Logit Transformed Response

- Logit-transform (0,1) response to values on real line
- Standard linear regression
- Ordinary least squares (OLS)
- Generalized least squares (GLS) → account for spatial dependence

Beta Distribution



- Very flexible distribution

→ accommodates various plant cover frequency distributions

$$f\left(y;p,q\right)=\frac{\Gamma\left(p+q\right)}{\Gamma\left(p\right)\Gamma\left(q\right)}y^{p-1}\left(1-y\right)^{q-1}$$

0 < *y* < 1; shape parameters *p*, *q* > 0, and gamma function *Γ(·)*

Beta regression (BR)

- *Ferrari & Cribari-Neto (2004) proposed new parameterization where:* $\mu = \dfrac{p}{p+q}$ $\qquad \phi = p+q$

  and introduced beta regression model:

$$g\left(\mu_i\right) = x_i^T \beta_i = \eta_i$$

- *Implemented in betareg R package (Cribari-Neto and Zeileis 2010) and PROC GLIMMIX in SAS*

Copula Model

- Copula: joins univariate marginal distributions into multivariate distribution function
- Multivariate Gaussian copula generalizes multivariate normal dependence structure to non-normal marginals
- A joint distribution function is

$$C\left(y;\Sigma\right)=\Phi_{\Sigma}\left[\Phi^{-1}\left\{F_1\left(y_1\right)\right\},\ldots,\Phi^{-1}\left\{F_n\left(y_n\right)\right\}\right]$$

Φ standard normal cdf

Φ_{Σ} multivariate normal cdf with correlation matrix Σ

Gaussian Copula Joint Density

- Differentiating the distribution funtion yields the joint density function:

$$c(y;\Sigma) = |\Sigma|^{-1/2} \exp\left(-\frac{1}{2}\mathbf{z}^T\left(\Sigma^{-1} - \boldsymbol{I}_n\right)\mathbf{z}\right)\prod_{i=1}^{n} f_i(y_i)$$

$$\mathbf{z} = \left[\Phi^{-1}\left\{F_1(y_1)\right\}, \ldots, \Phi^{-1}\left\{F_n(y_n)\right\}\right]^T$$

$\boldsymbol{I}_n$ denotes the *n* x *n* identity matrix

f_i is the marginal density of y_i

Σ determines the dependence structure

Spatial Gaussian Copula

- Spatial correlation matrix with exponential 'decay' parameter θ:

$$\Sigma_{ij}(\theta) = \begin{cases} \exp\left(-h_{ij}\theta\right), & i \neq j \\ 1, & i = j \end{cases}$$

- Spatial Gaussian copula brings non-normal distributions into Gaussian geostatistical framework (Madsen 2009)

- Obtain maximum likelihood estimates by numerically maximizing the log of expected likelihood with respect to $\boldsymbol{\beta}$, ϕ, and θ

Objectives

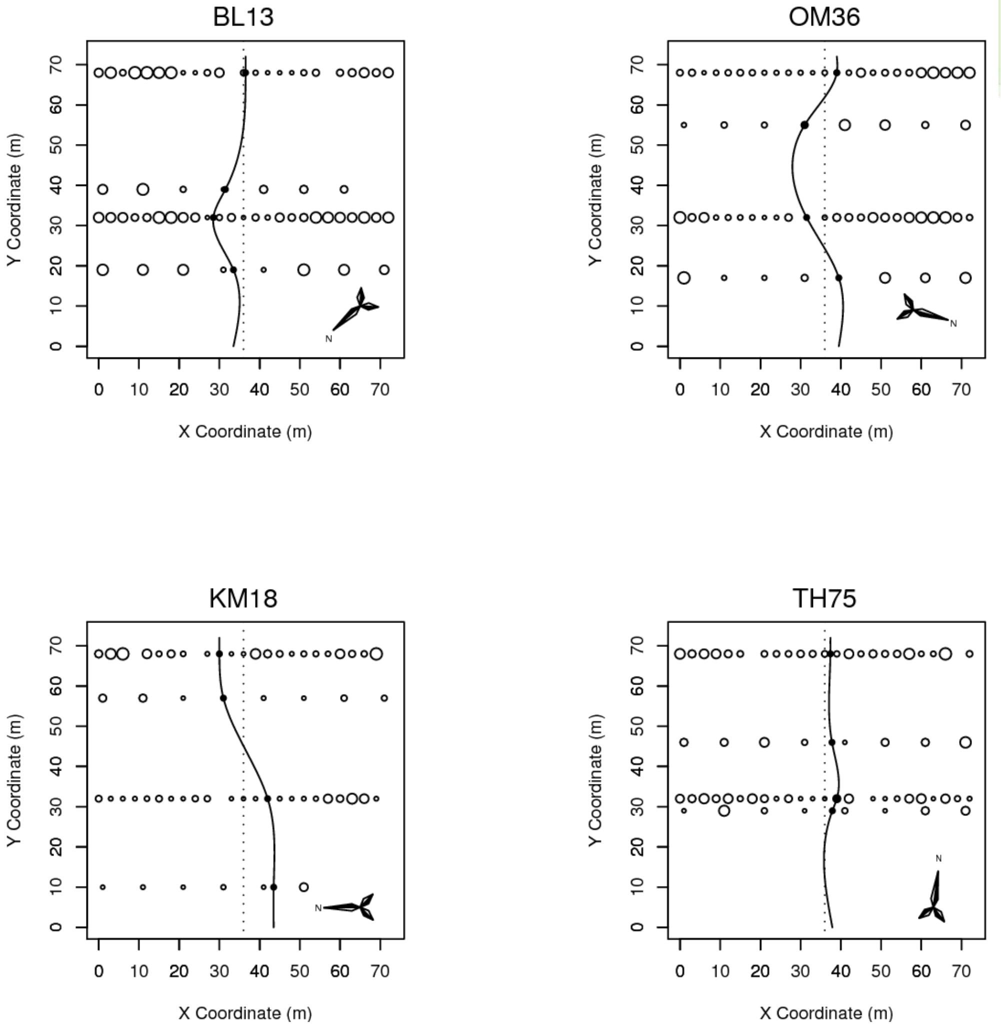
- *Case Study*: Model % shrub cover in riparian forests as function of topographic conditions and overstory vegetation characteristics using five modeling approaches:

  OLS, GLS, BR, BRdep, and COP

- *Simulation Study*: Evaluate the performance of five modeling approaches in terms of their parameter estimates

Case Study – Data

- 4 headwater streams in the Oregon Coast Range (Cissel et al. 2006, Marquardt 2010)
- Percent shrub cover
- Visually determined (nearest 5%) on 1 m x 1 m plots along transects (n=248)
- Distance to stream (DTS in m)
- Height above stream (HAS in m)
- Leaf area index (LAI)
- % slope, aspect



Case Study – Results

- Model with smallest Bayesian Information Criterion included DTS & LAI as covariates

- BR models had poor explanatory power (pseudo-$R^2 \leq 0.34$)

- *OLS & GLS models*: MSPE largest, negative bias

- *BR, BRdep & COP models*: MSPE slightly smaller, unbiased

Simulation Study – Data

- 500 data sets of size n=248 to mimic observed shrub data
- Simulated response from beta distribution with

$$\mu_i = \frac{1}{1+\exp\left(-\left(\beta_0 + \beta_1 DTS_i + \beta_2 LAI_i\right)\right)}$$

and $(\beta_0, \beta_1, \beta_2, \phi) = (-0.34, 0.037, -0.24, 2.6)$

- Spatial locations, DTS, and LAI agree with actual data from case study
- Spatial dependence in simulated response:
  - Simulate spatially dependent standard normal random variables Z
  - Covariance matrix based on exponential model with θ ranging from 0.01 (strong spatial dependence) to infinity (no spatial dependence)

$$\sum_{ij}(\theta) = \begin{cases} \exp\left(-h_{ij}\theta\right), & i \neq j \\ 1, & i = j \end{cases}$$

  - Transform Z's to beta response Y

Simulation Study – Methods

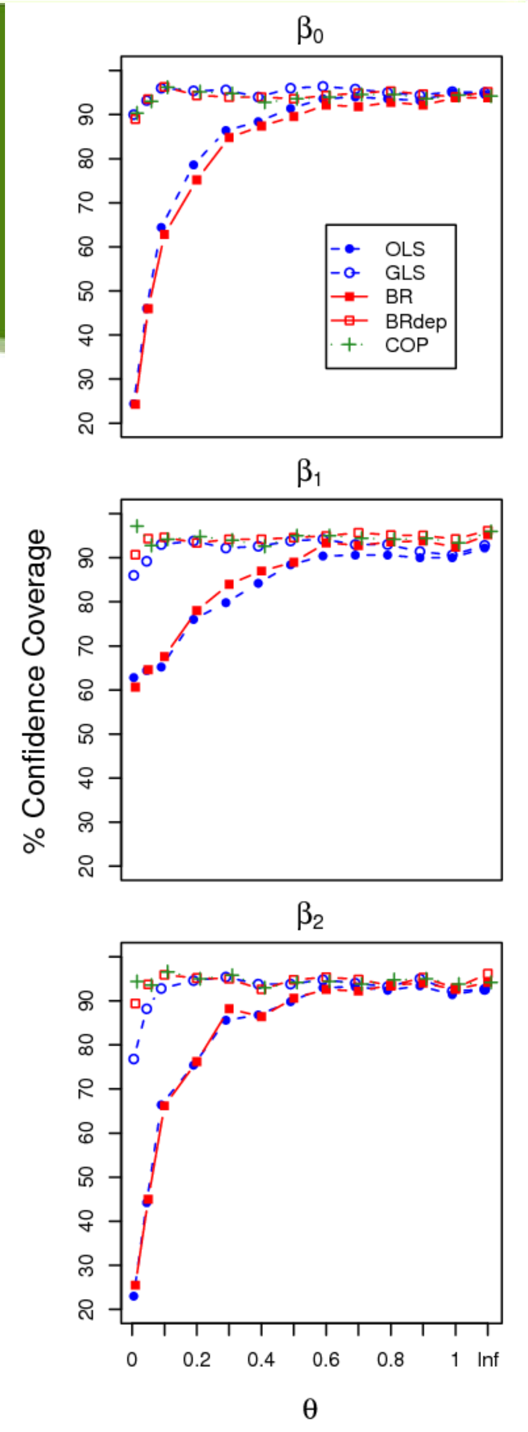
- Fit OLS, GLS, BR, BRdep, and COP models to 500 simulated data sets across range of θ values
- Calculate confidence coverage (CC) of parameters β_0, β_1, β_2

NOTE: OLS and GLS parameters can be expressed in terms of β_0, β_1, β_2 of the BR, BRdep, and COP models using Equation 15 in Espinheira et al. (2008)

Results – Simulation Study (cont'd)

- *OLS & BR*: poor CC for all β's when spatial dependence is strong (small θ)

- *GLS, BRdep & COP*:
  - 95% confidence coverage (CC) for all β's for $\theta \geq 0.1$
  - 77-95% CC for $\theta < 0.1$ with GLS having smallest CC



Conclusions

- Model fit is poor due to scale issues
- OLS and GLS resulted in biased model predictions → not recommended for modeling % shrub cover
- BR, BRdep, and COP provided unbiased predictions
- OLS & BR should not be used in the presence of spatial dependence
- When spatial dependence is strong, GLS, BRdep, and COP result in confidence coverage < 95%, with GLS performing worst
- BR and COP models should be extended to account for zero-inflation

Possible Future Applications

- Copula model not restricted to shrub cover
- Model % canopy cover
- Model crown ratio → currently working on a copula model for modeling crown ratio and tree height simultaneously

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Questions?

For more details see:

Eskelson, B.N.I., Madsen, L., Hagar, J., and H. Temesgen. *In press*. Estimating riparian understory vegetation cover with beta regression and copula models. Forest Science XX:xxx-xxx.