

# Marcy and Ted Take a Crack at Ratio Estimators

*A short essay about using ratio estimators to estimate timber volume.*

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*Abstract: Ratio estimators can be a useful tool for timber volume estimation. Few foresters have heard of, or are proficient with using ratio estimators. Here the author provides a short story to show one way that ratio estimators can be used in everyday forest management activities.*

Ted had arrived a little bit early this morning and was surprised to find Marcy already there and getting things together. “What are we up to today Marcy?” “Well Ted, I thought we would start marking that spruce-fir stand of timber today”. Ted shook his head a bit, that was going to be a lot of work because there were so many small stems there to mark and tally. “Marcy, maybe we could just put some cruise points in there to get the volume” Ted suggested. “Sorry Ted, but the Boss wants it marked and tallied. But I have a plan! I think we will use Ratio Estimators to get the job done a little bit easier and more efficiently” Marcy replied. “Ratio estimators? What’s that” Ted asked. Marcy told Ted that she would explain while they drive, first they needed to stop and get coffee and Travis would meet them at the coffee shop. “Travis? Why is he coming along?” Ted complained, since Travis had no cruising skills. Marcy just smiled and said “all will be revealed in good time, Ted. Just be patient”.

Marcy and Ted stopped at the coffee shop and there was Travis waiting for a ride. “Hey Travis, ready to do some work?” Marcy asked. Ted groaned and told Travis to sit in the back seat. Coffee in hand, off they went to start marking the sawlogs in the spruce-fir stand. When they arrived, they shook up their marking paint and packed it away in their cruising vests. Marcy handed Travis the leather tally book and told him he was the tally guy for the day.

“Here’s the plan, Ted and I will be marking the trees. We will make our best quick estimate of dbh and height and shout it back to Travis and he will tally it in the book” said Marcy. Travis rolling his eyes said, “how is that any different than any other time we have marked timber”? Marcy explained that usually they put a Biltmore stick or a D-tape on each marked tree and they measured most of the merchantable heights, but today it would be all quick eyeball estimates ... except that when we call out a tree, Travis will tally the tree using a “block of ten” system by dbh and log height then click the tally whacker to maintain a record of the total number of trees marked. It will be a systematic sample. “Travis, after a random start of 97 trees, every time you tally 200 additional trees from the tally whacker you will call out “Measure!” then we will carefully measure the tree, and you will record those measurements on the back page”. Travis gave her a puzzled look and said “OK, you’re the boss”!

| DBH | 1/2        | 1          | 1 1/2      | 2 | 2 1/2 | 3 | 3 1/2 |
|-----|------------|------------|------------|---|-------|---|-------|
| 6   | ☒☒☒☒☒☒☒☒☒☒ |            |            |   |       |   |       |
| 8   | ☒☒☒☒☒☒☒☒☒☒ | ☒☒☒☒☒☒☒☒☒☒ |            |   |       |   |       |
| 10  | ☒☒☒☒☒☒☒☒☒☒ | ☒☒☒☒☒☒☒☒☒☒ | ☒☒☒☒☒☒☒☒☒☒ |   |       |   |       |
| 12  |            |            |            |   |       |   |       |
| 14  |            |            |            |   |       |   |       |
| 16  |            |            |            |   |       |   |       |

Figure 1 Block of ten tally system on a paper tally sheet.

Marcy and Ted started in marking the timber. “Twelve by two and a half” and then “ten by one and a half” then “eight by one” they were calling out the quick eyeball estimated diameters at breast height and the merchantable heights in sixteen-foot logs. This went on for a while when suddenly Travis yells out “measure” to Marcy’s “10 by one and a half” call. So, Marcy put her paint gun down and measured the tree’s dbh with a logger’s tape to the nearest 0.1 inch. Then she measured out sixty-six feet and accurately measured the merchantable height to a six-inch top diameter. She called the measurements to Travis. On the back-page Travis recorded “RS, 10, 1.5; 10.4, 27” which would later be transcribed to a spreadsheet.

The marking and tallying continued throughout the morning. At lunch they sat down and dove into their sandwiches. They started to chat about the marking. “I can mark so much faster now that I don’t need to put the Biltmore stick on any trees” said Ted. Marcy said “that’s the point Ted, it’s all based on our guesses of the dbh and height. It’s not that important that we are accurate on our guesses, but it IS important that we be consistent in our guesses. When we finish marking the stand, I will put all the data into a spreadsheet application that I constructed.

Lunch was over and they went back to marking the timber. It took them three additional days to complete the marking and they had 20 measured stems to calculate ratios with. At that point, they had a very good estimate of the marked trees by DBH and height classes, but all with guesses instead of measurements, except for the twenty measured trees. Those guesses now needed to be corrected. For the twenty measured trees the difference between the guess and careful measurement was computed.

Marcy worked up the spreadsheet and then showed it to Ted. “What do you think”? Ted looked it over and gave a nod of approval.

**Table 1. Sample Tree Data**

| Sample |         | Guessed Sample Tree Data |        |                             | Measured Sample Tree Data |        |                             | Error<br>(m-Rg) <sup>2</sup> |
|--------|---------|--------------------------|--------|-----------------------------|---------------------------|--------|-----------------------------|------------------------------|
| Tree # | Species | DBH                      | Saw Ht | Saw Vol <sup>a</sup><br>(g) | DBH                       | Saw Ht | Saw Vol <sup>b</sup><br>(m) |                              |
| 1      | rs      | 10                       | 1.5    | 48                          | 10.4                      | 27     | 57.9                        | 21.71                        |
| 2      | bf      | 8                        | 1      | 24                          | 8.6                       | 18     | 27.4                        | 0.52                         |
| 3      | rs      | 12                       | 1.5    | 74                          | 11.8                      | 28     | 79.9                        | 4.96                         |
| 4      | rs      | 10                       | 1.5    | 48                          | 10.2                      | 25     | 52.4                        | 0.77                         |
| 5      | rs      | 12                       | 2      | 92                          | 12.4                      | 36     | 107.0                       | 23.78                        |
| 6      | rs      | 12                       | 2      | 92                          | 12.1                      | 38     | 104.9                       | 7.64                         |
| 7      | bf      | 10                       | 1.5    | 48                          | 10.7                      | 24     | 57.0                        | 13.81                        |
| 8      | rs      | 14                       | 2.5    | 153                         | 14.8                      | 43     | 182.7                       | 164.49                       |
| 9      | rs      | 12                       | 2      | 92                          | 11.5                      | 31     | 80.8                        | 452.43                       |
| 10     | rs      | 10                       | 1.5    | 48                          | 10.7                      | 28     | 63.6                        | 105.76                       |
| 11     | rs      | 8                        | 1      | 24                          | 7.7                       | 18     | 20.6                        | 36.27                        |
| 12     | bf      | 8                        | 1      | 24                          | 8.6                       | 19     | 28.5                        | 3.33                         |
| 13     | rs      | 8                        | 1      | 24                          | 8.2                       | 16     | 22.2                        | 19.57                        |
| 14     | rs      | 10                       | 1      | 36                          | 10.8                      | 18     | 47.6                        | 58.22                        |
| 15     | rs      | 8                        | 1      | 24                          | 8.1                       | 14     | 19.5                        | 51.12                        |
| 16     | bf      | 12                       | 1.5    | 74                          | 12.2                      | 27     | 84.2                        | 4.05                         |
| 17     | rs      | 14                       | 1.5    | 105                         | 14.8                      | 28     | 134.6                       | 325.49                       |
| 18     | bf      | 12                       | 2      | 92                          | 11.7                      | 30     | 82.3                        | 394.73                       |
| 19     | rs      | 10                       | 1      | 36                          | 10.3                      | 18     | 42.5                        | 6.58                         |
| 20     | rs      | 10                       | 1.5    | 48                          | 9.7                       | 25     | 46.5                        | 45.88                        |
|        |         | Sum                      |        | 1206                        |                           | Sum    | 1341.96                     | 1741.10                      |
|        |         | Mean                     |        | 60.3                        |                           | Mean   | 67.10                       |                              |

<sup>a</sup> From volume table

Ratio is 67.10/60.3 = 1.11

<sup>b</sup> From volume equation

Example of  $(m_i - Rg_i)^2$  for Tree #1 is  $(57.9 - (1.11 * 48))^2 = 21.71$

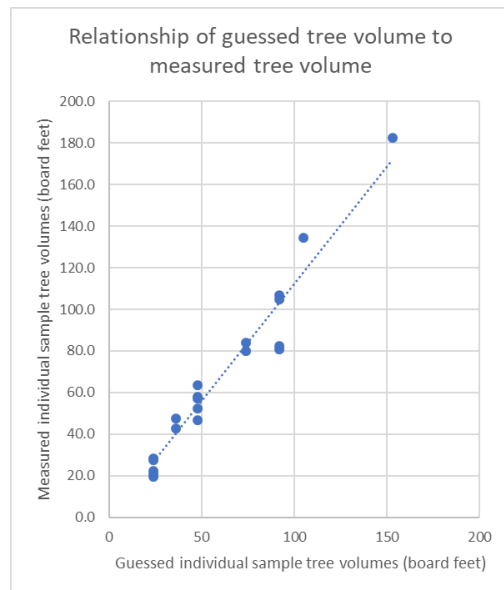
Marcy started with a graph of the guessed sample tree volumes on the horizontal axis and the measured sample tree volumes on the vertical axis. She explained “we can do this because as you can see there is a good relationship between the guessed and measured volumes. Our average ratio is 1.11, I know this because I divided the average measured volume by the average guessed volume which is  $67.1/60.3 = 1.11$ ”. “I see by the ratio that we tend to under-estimate spruce-fir volume by about 11% unless we measure the tree” said Ted. “Yes Ted, but it’s important not to change or adjust your estimates during the marking, because we want the guesses to remain consistent”.

Marcy explained “we KNOW that we marked 4,181 trees. The average guessed volume of all our marked trees is exactly 62.1 board feet per tree”. Now Marcy multiplied the average guessed volume of all 4,181 marked trees by the ratio to get the adjusted average overall tree volume ...

$$62.1 \text{ board feet} \times 1.11 = 68.93 \text{ board feet}$$

Marcy next explained that she was wondering how much confidence she could have in the adjusted average overall tree volume. She was able to find a variance estimator in Kim Iles<sup>1</sup> and in Steven Thompson’s<sup>2</sup> sampling books that looked like this ...

$$\widehat{SE}(\mu_r) = \sqrt{\left(\frac{(N - n)}{N}\right) * \frac{1}{n(n - 1)} * \sum m_i - Rg_i}$$



So now Marcy added in the specific information from this cruise and it looked as follows ...

$$\widehat{SE}(\mu_r) = \sqrt{\left(\frac{(4181 - 20)}{4181}\right) * \frac{1}{20(19)} * 1741} = 2.135$$

Also, since Marcy wanted a 95% confidence limit, she used a Student’s t for 19 degrees of freedom which was 2.093. Marcy calculated the confidence limits for the average tree volume as follows ...

$$\text{Upper} = 68.93 + (2.093 \times 2.135) = 73.400; \quad \text{Lower} = 68.93 - (2.093 \times 2.135) = 64.462$$

Ted now jumped in and said, “if we just multiply these numbers by the total number of trees, we can get total timber sale volumes as well, right?” So, they multiplied the average adjusted overall tree volume by the number of trees and got ...

<sup>1</sup> Iles, Kim 2003 “A Sampler of Inventory Topics” Kim Iles and Associates, Ltd., pages 347-351.  
<sup>2</sup> Thompson, Steven K. 2002 “Sampling” Wiley Series in Probability and Statistics, pages 68-71.

$$68.93 \times 4181 = 288,196 \text{ board feet}$$

of timber for the entire sale. The 95% confidence limits are calculated the same way ...

$$\text{Upper} = 73.400 \times 4181 = 306,885 \text{ board feet; Lower} = 64.462 \times 4181 = 269,515 \text{ board feet}$$

Ted studied the numbers carefully, then did the following math on a piece of scrap paper ...

$$\frac{306,885 - 288,196}{288,196} = 0.065 \text{ or } 6.5\%$$

Then he said “Or roughly the real volume will be our adjusted volume *plus or minus* 6.5 percent of the adjusted estimated volume 19 times out of 20. All we had to do to reach 6.5 percent was measure 20 trees. This is very similar to 3P which I learned in school, but doing it this way we didn’t have to deal with random numbers at each tree. We also can quickly tally using a fast and simple dot-dash block of 10 system instead of estimating and tallying the numerical volume of each individual tree. Later, that would allow us to do the volumes by DBH class or species as well, if we wished”. Marcy jumped in “There are some important differences from 3P cruising. We selected our measure trees with equal probability Ted, so we used the **Ratio of Means** estimator whereas, in 3P cruising the measure trees are selected by probability proportional to size, so in 3P cruising they use a **Mean of Ratios** estimator”.

Ted continued to look over the spreadsheet and said “I see that if we want to do this again, we can simply use this same spreadsheet, copy it and give it a new name, fill it in with the new data and all the calculations are set up already. Nice job boss! Let me buy you a coffee at the good place!”.



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